Schemes and Mechanisms of Neutrino Mixings (Oscillations) and a Solution of the Sun Neutrinos Deficit Problem

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Abstract

Three schemes of neutrino mixings (oscillations) are proposed. The problems of origin of angle mixings, with the law of energy-momentum conservation and disintegration of neutrino as wave pocket are solved. These two schemes belong to mass mixings schemes, where mixing angles and oscillation lengths are expressed via elements of mass matrix. The third scheme belongs to the charge mixings scheme, where mixing parameters are expressed via neutrino weak charges, as it takes place in the vector dominance model. Using experiments we must decide which of these schemes is realized indeed. Analysis of the resonance enhancement mechanism of neutrino oscillations in matter is performed. It is shown that there are no indications on existence of this effect. It is shown that the supposition that the neutrinos are Majorana particles is not confirmed by accelerator experiments. Then only mixings (oscillations) between Dirac neutrinos with different flavors without sterile neutrinos can be realized. Using all the present experimental data and the theoretical results the problem of Sun neutrinos deficit is analyzed. The conclusion is: the primary Sun ν_e neutrinos are converted into mixtures of three types of neutrinos ν_e, ν_μ, ν_τ in approximately equal quantities.

I. Introduction

In the old theory of neutrino oscillations [1, 2], constructed in the framework of Quantum Mechanics by analogy with the theory of $K^o, \bar{K}^o$ oscillation, it is supposed that mass eigenstates are ν_1, ν_2, ν_3 neutrino states but not physical neutrino states ν_e, ν_μ, ν_τ, and that the neutrinos ν_e, ν_μ, ν_τ are created as superpositions of ν_1, ν_2, ν_3 states. This meant that the ν_e, ν_μ, ν_τ neutrinos have no definite mass; i.e., their masses may vary depending on the ν_1, ν_2, ν_3 admixture in the ν_e, ν_μ, ν_τ states. It is clear that this picture is incorrect.

Originally, it was supposed [2] that these neutrino oscillations are real oscil-

lations; i.e., that real transition of electron neutrino ν_e into muon neutrino ν_μ (or tau neutrino ν_τ) takes place. Then the neutrino $x = \mu, \tau$ decays in electron neutrino plus something

$$\nu_x \rightarrow \nu_e + ..., \tag{1}$$

as a result, we get energy from vacuum, which equals the mass difference (if $m_{\nu_x} > m_{\nu_e}$)

$$\Delta E \sim m_{\nu_x} - m_{\nu_e}. \tag{2}$$

Then, again this electron neutrino transits into muon neutrino, which decays again and we get energy, and etc. **So we got a perpetuum mobile!** Obviously, the law of energy conservation cannot be fulfilled in this process. The only way to restore the law of energy conservation is to demand that this process is the virtual one. Then, these oscillations will be the virtual ones and they are described in the framework of the uncertainty relations. The correct theory of neutrino oscillations can be constructed only into the framework of the particle physics theory, where the conception of mass shell is present [3]-[6].

i) If the masses of the ν_e, ν_μ, ν_τ neutrinos are equal, then the real oscillation of the neutrinos will take place.

ii) If the masses of the ν_e, ν_μ, ν_τ are not equal, then the virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasi-elastic interactions, in order to undergo transition to the mass shell of other appropriate neutrinos by analogy with $\gamma - \rho^o$ transition in the vector meson dominance model.

At first we consider three schemes of neutrino mixings (oscillations), then analyse main mechanisms of neutrino mixings (oscillations) and then come to a solution of the Sun neutrinos deficit problem using all available experimental data on neutrino mixings.

II. Schemes of Neutrino Mixings (Oscillations)

The mass matrix of ν_e and ν_μ neutrinos has the following form:

$$\left(\begin{array}{cc} m_{\nu_e} & 0 \\ 0 & m_{\nu_\mu} \end{array} \right). \tag{3}$$

Due to the presence of the interaction violating the lepton numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed into

the following nondiagonal matrix (CP is conserved):

$$\begin{pmatrix} m_{\nu_e} & m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu} \end{pmatrix}, \tag{4}$$

then the lagrangian of mass of the neutrinos takes the following form ($\nu \equiv \nu_L$):

$$\begin{array}{c} \mathcal{L}_M = -\frac{1}{2}\left[m_{\nu_e}\bar{\nu}_e\nu_e + m_{\nu_\mu}\bar{\nu}_\mu\nu_\mu + m_{\nu_e\nu_\mu}(\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_e)\right] \equiv \\ \equiv -\frac{1}{2}(\bar{\nu}_e, \bar{\nu}_\mu)\begin{pmatrix} m_{\nu_e} & m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu} \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \end{array}, \tag{5}$$

which is diagonalized by turning through the angle θ and (see ref. in [2]) and then this lagrangian (5) transforms into the following one:

$$\mathcal{L}_M = -\frac{1}{2}\left[m_1\bar{\nu}_1\nu_1 + m_2\bar{\nu}_2\nu_2\right], \tag{6}$$

where

$$m_{1,2} = \frac{1}{2}\left[(m_{\nu_e} + m_{\nu_\mu}) \pm \left((m_{\nu_e} - m_{\nu_\mu})^2 + 4m^2_{\nu_\mu\nu_e}\right)^{1/2}\right],$$

and angle θ is determined by the following expression:

$$tg2\theta = \frac{2m_{\nu_e\nu_\mu}}{(m_{\nu_\mu} - m_{\nu_e})}, \tag{7}$$

$$\begin{array}{c} \nu_e = cos\theta\nu_1 + sin\theta\nu_2, \\ \nu_\mu = -sin\theta\nu_1 + cos\theta\nu_2. \end{array} \tag{8}$$

From Exp.(7) one can see that if $m_{\nu_e} = m_{\nu_\mu}$, then the mixing angle is equal to $\pi/4$ independently of the value of $m_{\nu_e\nu_\mu}$:

$$sin^2 2\theta = \frac{(2m_{\nu_e\nu_\mu})^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + (2m_{\nu_e\nu_\mu})^2}, \tag{9}$$

$$\begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix}.$$

It is interesting to remark that expression (9) can be obtained from the Breit-Wigner distribution [7]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}, \tag{10}$$

by using the following substitutions:

$$E = m_{\nu_e}, \;\; E_0 = m_{\nu_\mu}, \;\; \Gamma/2 = 2m_{\nu_e,\nu_\mu},$$

where $\Gamma/2 \equiv W(...)$ is a width of $\nu_e \to \nu_\mu$ transition, then we can use a standard method [4, 8] for calculating this value.

We can also see that there are two cases of ν_e, ν_μ transitions (oscillations) [4]-[6].

1. If we consider the transition of ν_e into ν_μ particle, then

$$sin^2 2\theta \cong \frac{4m^2_{\nu_e,\nu_\mu}}{(m_{\nu_e} - m_{\nu_\mu})^2 + 4m^2_{\nu_e,\nu_\mu}}, \tag{11}$$

How can we understand this $\nu_e \to \nu_\mu$ transition?

If $2m_{\nu_e,\nu_\mu} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of ν_e particle is m_{ν_e} and this mass is distributed by $sin^2 2\theta$ (or by the Breit-Wigner formula) and the probability of the $\nu_e \to \nu_\mu$ transition differs from zero and it is defined by masses of ν_e and ν_μ particles and m_{ν_e,ν_μ}, which is computed in the framework of the standard method, as pointed out above.

Another interpretation of Exp. (11) is: If m_{ν_e,ν_μ} differs from zero then Exp. (11) gives probability of $\nu_e \leftrightarrow \nu_\mu$ transitions. If $m_{\nu_e,\nu_\mu} = 0$, then the $\nu_e \leftrightarrow \nu_\mu$ transitions are forbidden.

So, this is a solution of the problem of the of mixing angle origin in the theory of vacuum oscillations.

In this case the probability of $\nu_e \to \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \to \nu_\mu, t) = sin^2 2\theta sin^2 \left[\pi t \frac{\mid m^2_{\nu_1} - m^2_{\nu_2} \mid}{2p_{\nu_e}}\right], \tag{12}$$

$$L_o = 2\pi \frac{2p}{\mid m^2_2 - m^2_1 \mid},$$

where p_{ν_e} is a momentum of ν_e neutrino and L_o is the length of neutrino oscillations.

2. If we consider the virtual transition of ν_e into ν_μ neutrino at $m_{\nu_e} = m_{\nu_\mu}$ (i.e., without changing the mass shell), then

$$tg 2\theta = \infty, \tag{13}$$

$\theta = \pi/4$, and

$$sin^2 2\theta = 1. \tag{14}$$

In this case the probability of the $\nu_e \to \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \to \nu_\mu, t) = sin^2\left[\pi t \frac{4m^2_{\nu_e,\nu_\mu}}{2p_a}\right]. \tag{15}$$

In order to make these virtual oscillations real, their participation in quasi-elastic interactions is necessary for the transitions to their own mass shells [8].

It is clear that the $\nu_e \to \nu_\mu$ transition is a dynamical process.

3. The third type of transitions (oscillations) can be realized by mixings of the fields (neutrinos) by analogy with the vector dominance model ($\gamma - \rho^o$ and $Z^o - \gamma$ mixings) in a way as it takes place in the particle physics. Since the weak couple constants $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of ν_e, ν_μ, ν_τ neutrinos are nearly equal in reality, i.e., $g_{\nu_e} \simeq g_{\nu_\mu} \simeq g_{\nu_\tau}$ the angle mixings are nearly maximal:

$$sin\theta_{\nu_e\nu_\mu} \simeq \frac{g_{\nu_e}}{\sqrt{g^2_{\nu_e} + g^2_{\nu_\mu}}} = \frac{1}{\sqrt{2}} \simeq sin\theta_{\nu_e\nu_\tau} \simeq sin\theta_{\nu_\mu\nu_\tau}. \tag{16}$$

Therefore, if the masses of these neutrinos are equal (which is hardly probable), then transitions between neutrinos will be real and if the masses of these neutrinos are not equal, then transitions between neutrinos will be virtual by analogy with $\gamma - \rho^o$ transitions. In this approach we can hardly see neutrino oscillations, though it is possible, in principle.

Probably, a more realistic type of neutrino transitions is the third one since oscillations can arise by a dynamics. Exactly the third type of mixings is realized through dynamical charges $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of neutrinos but not through neutrino masses. Nevertheless, we must get an answer to the question: which of the above three types of neutrino oscillations is realized in the Nature?

III. Mechanisms of Neutrino Oscillations

III.1. Impossibility of Resonance Enhancement of Neutrino Oscillations in Matter

Via three different approaches: by using mass Lagrangian [5], [9,10], by using the Dirac equation [5, 11], and by using the operator formalism [12], the author of this work has discussed the problem of the mass generation in the standard weak interactions and has come to a conclusion that the standard weak interaction

cannot generate masses of fermions since the right-handed components of fermions do not participate in these interactions. It is also shown [13] that the equation for Green function of the weak-interacting fermions (neutrinos) in the matter coincides with the equation for Green function of fermions in vacuum and the law of conservation of the energy and the momentum of neutrino in matter will be fulfilled [12] only if the energy W of polarization of matter by the neutrino or the corresponding term in Wolfenstein equation [14], is zero (it means that neutrinos cannot generate permanent polarization of matter). These results lead to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

The simplest method to prove the absence of the resonance enhancement of neutrino oscillations in matter is:

If we put an electrical (e) (or strong (g)) charged particle a in vacuum, there arises polarization of vacuum. Since the field around particle a is spherically symmetrical, the polarization must also be spherically symmetrical. Then the particle will be left at rest and the law of energy and momentum conservation is fulfilled.

If we put a weakly (g_W) interacting particle b (a neutrino) in vacuum, then since the field around the particle has a left-right asymmetry (weak interactions are left-handed interactions with respect to the spin direction [15, 16]), polarization of vacuum must be nonsymmetrical; i.e., on the left side there arises maximal polarization and on the right there is zero polarization. Since polarization of the vacuum is asymmetrical, there arises asymmetrical interaction of the particle (the neutrino) with vacuum and the particle cannot be at rest and will be accelerated. Then the law of energy momentum conservation will be violated. The only way to fulfil the law of energy and momentum conservation is to demand that polarization of vacuum be absent in the weak interactions. The same situation will take place in matter. It is necessary to remark that for the above-considered proof it is sufficient to know that the field around the weakly interacting particle is asymmetrical (and it is not necessary to know the precise form of this field). It is necessary also to remark that the Super-Kamiokande data on day-night asymmetry [17] is

$$A = (D-N)/(\frac{1}{2}(D+N)) = -0.021 \pm 0.020(stat) + 0.013(-0.012)(syst). \quad (17)$$

and it does not leave hope on possibility of the resonance enhancement of neutrino oscillations in matter.

In means that the forward scattering amplitude of the weak interactions has

specific behavior.

In the Sun neutrino experiments, it is impossible to distinguish muon and tau neutrinos since energies of the Sun neutrinos are less than the threshold energy of muon and tau lepton productions. However, we can distinguish these neutrinos on the Earth-long- baseline accelerator neutrino experiments with neutrino energies higher than the threshold energy of muon and tau lepton creations. Probably, such experiments put an end to the problem of resonance enhancement of neutrino oscillations in matter (see the strong proof absence of this effect given above).

III.2. Majorana Neutrino Oscillations

At present it is supposed [18] that the neutrino oscillations can be connected with Majorana neutrino oscillations. It will be shown that we cannot put Majorana neutrinos in the standard Dirac theory. It means that in experiments the Majorana neutrino oscillations cannot be observed.

Majorana fermion in Dirac representation has the following form [1, 2, 19]:

$$\chi^M = \frac{1}{2}[\Psi(x) + \eta_C \Psi^C(x)], \tag{18}$$
$$\Psi^C(x) \to \eta_C C \bar{\Psi}^T(x),$$

where η_C is a phase, C is a charge conjunction, T is a transposition.

In the standard theory all fermion are Dirac particles and this theory is gauge invariance one. Then the supposition that neutrino is a Majorana particle is a right violation of gauge invariance [6, 20].

If, in spite of the above arguments, we put Majorana neutrinos in the standard theory, there appears two possibilities:

1. If the second component of Majorana neutrino in Eq.(18) is antineutrino, then the neutrinoless double beta decay will take place, neutrino will transit into antineutrino while oscillations and then in accelerator experiments we must see the following reactions:

$$\chi_l + A(Z) \to l^- + A(Z+1), \tag{19}$$

with relative probability 1/2 and

$$\chi_l + A(Z) \to l^+ + A(Z-1), \tag{20}$$

with the same relative probability (where $l = e, \mu, \tau$), since Majorana neutrinos are superpositions of Dirac neutrinos and antineutrinos.

2. If the second component of Majorana neutrino in Eq.(18) is a sterile neutrino, then the neutrinoless double beta decay cannot exist, since the second component has incorrect spirality but we can see neutrino disappears at oscillations and then in accelerator experiments we must see the following reactions:

$$\chi_l + A(Z) \to l^- + A(Z+1), \quad l = e, \mu, \tau, \tag{21}$$

with relative probability 1/2. The second sterile components of Majorana neutrinos in Eq.(18) do not take part in the standard weak interactions. Obviously, all the available accelerator experimental data [21] do not confirm these predictions; therefore we cannot consider this mechanism as a realistic one for neutrino oscillations. Then, in principle, transition of the Dirac neutrinos into of the Majorana neutrinos can be, but it is only possible at full violation of the lepton numbers, i.e., at the Grand Unification scales ($T > 10^{30}$ y.).

III.3. Mixings (Oscillations) of Flavor Neutrinos

In the work [22] Z. Maki et al. and B. Pontecorvo in [23] supposed that there could exist transitions between aromatic neutrinos ν_e, ν_μ. Afterwards ν_τ was found and then ν_e, ν_μ, ν_τ transitions could be possible. It is necessary to remark that only this scheme of oscillations is realistic for neutrino oscillations. The expressions, which described neutrino oscillations in this case are given above in expressions (11)-(17).

IV. A Solution of the Sun Neutrinos Deficit Problem

Before consideration of this problem it is important to consider the necessary data.

The value of the Sun neutrinos flow measured (through elastic scattering) on SNO [24] is in good agreement with the same value measured in Super-Kamiokande [25].

Ratio of ν_e flow measured on SNO (CC) to the same flow computed in the frame work of SSM [26] ($E_\nu > 6.0 MeV$) is:

$$\frac{\phi^{CC}_{SNO}}{\phi_{SSM2000}} = 0.35 \pm 0.02. \tag{22}$$

This value is in good agreement with the same value of ν_e relative neutrinos flow measured on Homestake (CC) [27] for energy threshold $E_\nu = 0,814 MeV$.

$$\frac{\Phi^{exp}}{\Phi^{SSM2000}} = 0.34 \pm 0.03. \tag{23}$$

From these data we can come to a conclusion that the angle mixing for the Sun ν_e neutrinos does not depend on neutrino energy thresholds. Now it is necessary to know the value of this angle mixing $\theta_{\nu_e \nu_\mu}$. Estimation of the value of this angle can be extracted from KamLAND [28] data and it is:

$$sin^2 \theta_{\nu_e \nu_\mu} \cong 1.0, \quad \theta \cong \frac{\pi}{4}, \tag{24}$$

The angle mixing for vacuum $\nu_\mu \to \nu_\tau$ transitions obtained on Super-Kamiokande [29] for atmospheric neutrinos is:

$$sin^2 2\theta_{\nu_\mu \nu_\tau} \cong 1, \quad \theta \cong \frac{\pi}{4}. \tag{25}$$

Now we can estimate the third angle mixing for $\nu_e \to \nu_\tau$ transitions. The full flow neutrinos obtained on SNO [24] is:

$$\phi_{SNO}(\nu_x) = (5.09 \pm 0.44) \times 10^6 cm^{-2} c^{-1}. \tag{26}$$

This result is in good agreement with the following prediction of the Standard Sun Model [26]:

$$\phi_{SNO}(\nu_x) = (5.05 + 1.01(-0.81)) \times 10^6 cm^{-2} c^{-1}. \tag{27}$$

This agreement is an indication on absence of neutrino disappearance, i. e. the sterile neutrino hypothesis is not confirmed.

The SNO experiment has measured only the summary flow of $\mu_\mu + \nu_\tau$ neutrinos. Now we must separate these flows. For this purpose we can use neutrino mixing parameters obtained on Super-Kamiokande for atmospheric neutrinos (25), on KamLAND (24) for reactor antineutrinos and also the following flow of the Sun electron neutrinos measured on SNO [24]:

$$\phi^{CC}_{SNO}(\nu_e) = (1.76 \pm 0.11) \cdot 10^6 cm^{-2} s^{-1}. \tag{28}$$

In the subsequent considerations we will take into account that in the Sun neutrino experiments where the measured neutrinos with energy up to 15 MeV, and that the Earth orbit is not a circular one i.e., the measured values for the Sun neutrinos are average values.

So, since the obtained vacuum angle mixing for $\nu_e \to \nu_\mu$ is close to the maximal angle then the sum $\nu_e+\nu_\mu$ Sun neutrino flow is a doubled flow of ν_e Sun neutrinos

$$\phi_{SNO}(\nu_e + \nu_\mu) \cong (3.52 \pm 0.22) \cdot 10^6 cm^{-2} s^{-1}. \tag{29}$$

Then the following remainder of the Sun neutrinos flow is the flow of ν_τ Sun neutrinos:

$$\phi_{SNO}(\nu_\tau) \cong \phi_{SNO}(\nu_x) - \phi_{SNO}(\nu_e + \nu_\mu) \cong (1.57 \pm 0.49) \cdot 10^6 cm^{-2} c^{-1}. \tag{30}$$

It is about one third of the primary Sun neutrino flow. Since the angle mixings of $\theta_{\nu_e \nu_\mu}$ and $\theta_{\nu_\mu \nu_\tau}$ neutrinos are close to maximal angles, then we have no reason to suppose that the $\theta_{\nu_e \nu_\mu}$ angle mixing distinctly differs from the maximal one. So we come to the following conclusion: on way to the Earth the primary Sun ν_e neutrinos transit in the mixture of electron, muon and tau neutrinos in approximately equal quantities:

$$\phi_{Sun}(\nu_e) \to \phi_{Sun}(\nu) \cong \frac{1}{3}\phi(\nu_e) + \frac{1}{3}\phi(\nu_\mu) + \frac{1}{3}\phi(\nu_\tau). \tag{31}$$

This is a decision of the Sun neutrinos deficit problem at the qualitative level i.e., the vacuum mixing angles are near to the maximal ones and there are no serious indication on existence of sterile neutrinos. Besides there are no experimental indication on realization of the mechanism of resonance enhancement of neutrino oscillations in matter. It is necessary, in subsequent long baseline experiments on the Earth, to precise all neutrino angle mixings and squared mass differences of neutrinos for their using for precise solution of the Sun neutrinos deficit problem. The problem about existence or absence of neutrino oscillations is very important.

IV. Conclusion

Three schemes of neutrino mixings (oscillations) has been proposed. The first scheme is a development of the standard schemes in the framework of particle physics, where problems of origin of angle mixings, with the law of energy-momentum conservation and disintegration of neutrino as wave pocket are solved. In the second scheme the angle mixing is maximal. These two schemes belong to mass mixings schemes, where mixing angles and oscillation lengths are expressed via elements of mass matrix. The third scheme belongs to the charge mixings scheme, where mixing parameters are expressed via neutrino weak charges, as it takes place in the vector dominance model. Using experiments we must decide

which of these schemes is realized indeed. Analysis of the resonance enhancement mechanism of neutrino oscillations in matter is performed. It is shown that there are no indications on existence of this effect. It is shown that the supposition that the neutrinos are Majorana particles has not been confirmed by accelerator experiments. Then only mixings (oscillations) between Dirac neutrinos with different flavors without sterile neutrinos can be realized. Using all the present experimental data and the theoretical results the problem of Sun neutrinos deficit has been analyzed. The conclusion is: the primary Sun ν_e neutrinos are converted into mixtures of three types of neutrinos ν_e, ν_μ, ν_τ in approximately equal quantities.

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